On Conditions for Morphogenetic Diversity of Multicellular Organisms

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1 Introduction

In a multicellular organism, a single cell—an egg—or a group of cells develops into a certain pattern with a variety of cell types. The variety of cell types are created through cell differentiation; the differentiation starts from an initial type, and then the initial type changes into several intermediate types before differentiating into the final type.

Theoretical study of cell differentiation and morphogenesis was pioneered by Alan Turing, who showed that a reaction–diffusion system can produce an inhomogeneous stable pattern [4]. Turing’s theory provides a basis of dynamical system for morphogenesis and potentiality of cell differentiation. Embryogenesis with increases of cell numbers was, however, not studied.

By considering Turing’s study and intracellular dynamics, together with the cell division process to increase the cell numbers, Kaneko and Yomo have proposed isologous diversification [2, 3]. This allows the spontaneous cell differentiation through cell division processes and cell–cell interactions.

These studies provide a basis for morphogenetic diversity of multicellular organisms. However, relevance of proliferation rates and transition rates between cell types to morphogenetic diversity has not been studied. In this paper, we shall answer this question by constructing the model based on probabilistic Lindenmayer system with interaction and by using quantifier elimination (abbreviated to QE) [6].

2 Model

We shall present a model of a multicellular organism using probabilistic Lindenmayer system. See Fig. 1 for a schematic illustration of our model. We assume that cell differentiation starts from an initial type $I_0$, and then the cell differentiates to several intermediate types $I_1 \rightarrow I_2 \rightarrow \ldots \rightarrow I_{n-1}$ before it differentiates into the final type $I_n$. The proliferation ($I_i \rightarrow I_i I_i$) and transition ($I_i \rightarrow I_{i+1}$) rates of cell type $i$ ($0 \leq i \leq n$) are defined as follows:

$$I_i \rightarrow \begin{cases} I_i I_i & p_{i,i} \\ I_{i+1} & 1-p_{i,i} \\ I_i & p_{i,i+1} - p_{i,i+1} \end{cases} \quad (0 \leq i < n), \quad I_n \rightarrow \begin{cases} I_n I_n & p_{n,n} \\ I_n & 1-p_{n,n} \end{cases}$$
In addition to the productions above, we adopt a production: $I_iI_j \rightarrow I_iI_{i+1} \cdots I_{j-1}I_j$ ($j > i + 1$), which guarantees the contiguity of cell types. The production above originates from the intercalary regeneration in cockroach legs, planarian or salamander limb blastema [1, 5].

3 Results and Discussion

Now, we calculate the transition matrix $M$ of the two contiguous cell types $I_iI_i$, $I_iI_{i+1}$, $I_{i+1}I_i$ ($0 \leq i < n$), which enables us to estimate the composition of $I_iI_k$ ($k = \ell - 1, \ell, \ell + 1$) at step $m$. If one starts with $I_0I_0$, the composition at step $m$ can be calculated by the following formula:

$$(1, 0, 0, \ldots)M^m$$

Here, we have studied the case of $n = 2$, showing the existence of three cell types. One of necessary conditions that $I_0I_0, I_0I_1, I_1I_0, I_1I_1, I_1I_2, I_2I_1, I_2I_2$ are well mingled as $m$ approaches infinity is obtained:

$$1 + 2p_{0,0} - p_{0,1} > 1$$

Under these conditions and $p_{2,2} = 0$, we have calculated the most diverse composition: $(I_0I_0, I_0I_1, I_1I_0, I_1I_1, I_1I_2, I_2I_1, I_2I_2)$ by $QE$ method. By the calculation, we have obtained the following optimized composition:

$$(I_0I_0, I_0I_1, I_1I_0, I_1I_1, I_1I_2, I_2I_1, I_2I_2) = (1, 2/3, 2/3, 1, 1/12(\sqrt{129} - 3), 1/12(\sqrt{129} - 3), 1),$$

on condition that the values of $p_{0,0}, p_{0,1}, p_{1,1}, p_{1,2}$ are nearly 0. Notice again that the condition for the most diverse composition is that $p_{0,0}, p_{0,1}, p_{1,1}, p_{1,2}$ are nearly 0. This fact leads us to recognize that the permanent appearance of $I_0I_1I_2$ domains is important during the proliferation and transition processes.

References


